

73. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{dt} = 6.0t^2 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = 12t$$

with SI units understood. These expressions are used in the parts that follow.

(a) Using the definition of average velocity, Eq. 2-2, we find

$$v_{\text{avg}} = \frac{x(2) - x(1)}{2.0 - 1.0} = \frac{2(2)^3 - 2(1)^3}{1.0} = 14 \text{ m/s} .$$

(b) The average acceleration is defined by Eq. 2-7:

$$a_{\text{avg}} = \frac{v(2) - v(1)}{2.0 - 1.0} = \frac{6(2)^2 - 6(1)^2}{1.0} = 18 \text{ m/s}^2 .$$

(c) The value of $v(t)$ when $t = 1.0$ s is $v(1) = 6(1)^2 = 6.0$ m/s.

(d) The value of $a(t)$ when $t = 1.0$ s is $a(1) = 12(1) = 12$ m/s².

(e) The value of $v(t)$ when $t = 2.0$ s is $v(2) = 6(2)^2 = 24$ m/s.

(f) The value of $a(t)$ when $t = 2.0$ s is $a(2) = 12(2) = 24$ m/s².

(g) We don't expect average values of a quantity, say, heights of trees, to equal any particular height for a specific tree, but we are sometimes surprised at the different kinds of averaging that can be performed. Now, the acceleration is a linear function (of time) so its average as defined by Eq. 2-7 is, not surprisingly, equal to the arithmetic average of its $a(1)$ and $a(2)$ values. The velocity is not a linear function so the result of part (a) is not equal to the arithmetic average of parts (c) and (e) (although it is fairly close). This reminds us that the calculus-based definition of the average a function (equivalent to Eq. 2-2 for v_{avg}) is not the same as the simple idea of an arithmetic average of two numbers; in other words,

$$\frac{1}{t' - t} \int_t^{t'} f(\tau) d\tau \neq \frac{f(t') + f(t)}{2}$$

except in very special cases (like with linear functions).

(h) The graphs are shown below, $x(t)$ on the left and $v(t)$ on the right. SI units are understood. We do not show the tangent lines (representing instantaneous slope values) at $t = 1$ and $t = 2$, but we do show line segments representing the average quantities computed in parts (a) and (b).

